MODELS FOR THE MIXED COLLECTORS BY USING REGRESSION ANALYSIS

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ABSTRACT
In this study, we develop some mathematical models for the enrichment of copper ore of North Waziristan. Seven different variables were considered. The most important flotation reagents i.e. type and dosage of propylxanthate, ($X_1$ g/ton), pH ($X_2$), Sodium Cyanide, ($X_3$ g/ton), Sodium sulphide ($X_4$ g/ton) frother ($X_5$ g/ton) pulp density ($X_6$, % wt/ Vol) conditioning time ($X_7$, minute) were examined. Flotation parameter of copper was upgraded from 0.9 to 20% during the stepwise optimization. Mixed collector was examined to get the maximum grade of copper. Several mathematical models were developed by applying ordinary least squares (OLS) method for regression analysis and adopted best subset modeling procedure. The model given in equation (5) which includes the process parameters propyl xanthate, Sodium Cyanide and conditioning time is statistically significant.

INTRODUCTION:
Copper is one of the most essential mineral for modern industry. It is normally a prosperity metal, which is used when expansion of electricity network takes place, but it is also an essential metal for weaponry and to a lessor extent, in foundry and chemical plants. World mine production of copper increased by 45% (to 13.20 million metric tones of contained copper) between 1990 and 2000, at an annual rate of growth of 3.8%.

In the tribal area of North Waziristan, Federally Administered Tribal Area (FATA) carried out exploration work and 1.5 million tons of estimated reserve (Badshah, 1983; Khan, 1984; Badshah, 1985) of copper ore in ShinKai and Degan area of North Waziristan were found.

Experimental work was carried out by the Department of Mining Engineering NWFP University of Engineering and Technology Peshawar. (Khan, M.M., 2005; Wang, Z., 1996).

Federally Administered Tribal Areas (FATA) Development Corporation (Pakistan) carried out exploration and reported an estimate of 122 million tones reserves of copper ore in Shinkai and Degan area of North Waziristan. It is envisaged that statistical approach based on the was reported earlier to predict the most optimal conditions for flotation of North Waziristan copper deposits. Cilek E.C., 2004, has developed a classical first-order kinetic model, combined with a properly built statistical model. A statistical approach has been reported by Barbo, M. and M. Baro and Piga, L., 1999 to evaluate the lead zinc selectivity by various collectors types in flotation.

Strategy Modeling
It is difficult to fit and test all the possible regression models involving seven variables therefore the best subset procedure was used to select models involving one, two, three, four, five and six variables. The best subset procedure (Minitab) produced the following thirteen models two in each subset and the full model for grade of copper.

Models with one variable are:

\[ M_1: Y_0 = \beta_0 + \beta_1 X_1 \]
\[ M_2: Y_0 = \beta_0 + \beta_2 X_4 \]

Models with two variables are:

\[ M_3: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_7 \]
\[ M_4: Y_0 = \beta_0 + \beta_3 X_3 + \beta_2 X_7 \]

Models with three variables are:

\[ M_5: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 \]
\[ M_6: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 \]

Models with four variable are:

\[ M_7: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 \]
\[ M_8: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 \]

Models with five variable are:

\[ M_9: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 + \beta_6 X_7 \]
\[ M_{10}: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 + \beta_6 X_7 + \beta_7 X_7 \]

Models with six variable are:

\[ M_{11}: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 + \beta_6 X_7 + \beta_7 X_7 \]
\[ M_{12}: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 + \beta_6 X_7 + \beta_7 X_7 \]

Models with seven variable are:

\[ M_{13}: Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_7 + \beta_4 X_7 + \beta_5 X_7 + \beta_6 X_7 + \beta_7 X_7 \]
RESULTS AND DISCUSSION

The regression equations for single predictors for recovery of copper obtained form least square analysis are as follows:

\[ Y_G = 14.0 + 0.0854 X_1 \ldots \ldots R^2 = 24.3\% \quad \text{(1)} \]

\[ Y_G = 14.5 + 0.0374 X_4 \ldots \ldots R^2 = 20.7\% \quad \text{(2)} \]

The \( R^2 \) show that the first equation explained 24.3\% of the variation and the second equation explained 20.7\% variation in the grade of copper using flotation equation process.

**Note:** The \( R^2 \) is not seen in the article.

Among the twenty-one models in the subset with two predictor variables, the two best regression equations involving two predictor variables are:

\[ Y_G = 20.5 + 0.0518 X_4 - 0.599 X_7 \quad R^2 = 46.1\% \quad \text{(3)} \]

\[ Y_G = 19.3 + 0.104 X_3 - 0.527 X_7 \quad R^2 = 45.1\% \quad \text{(4)} \]

The equations involving \( X_4 \) and \( X_7 \) explained 46.1\% and the equation involving \( X_3 \) and \( X_7 \) explained 45.1\% of the variation in the grade of copper.

Response surfaces were developed for the variables involved in the above two equations.

The best subset program picked the following two best regression equations involving three predictor variables among the 55, 3-variable models.

\[ Y_G = 16.5 + 0.0216 X_1 + 0.0448 X_4 - 0.598 X_7 \ldots \ldots \quad \text{(5)} \]

\[ Y_G = 15.9 + 0.0191 X_1 + 0.0859 X_3 - 0.525 X_7 \ldots \ldots \quad \text{(6)} \]

The two equations explain 58.6\% and 54.3\% of total variation in grade of copper. Among the next subset with four predictors, the following two best regression equations were selected by the program:

\[ Y_G = 20.4 + 0.0216 X_1 + 0.0411 X_4 - 0.141 X_6 - 0.570 X_7 \quad \text{(7)} \]

\[ Y_G = 20.2 + 0.0193 X_1 + 0.0601 X_3 - 0.0437 X_5 - 0.626 X_7 \quad \text{(8)} \]

Equation (7) explained 62.7\% of the variation in the data however; the contribution of variable \( X_6 \) is not important in this model. Equation (8) explained 61.1\% of the variation in the data. All variables in this model are collectively important so this is a good fit model for grade of copper.

The following two best regression equation involving five predictor variables were selected by the program:

\[ Y_G = 20.5 + 0.0194 X_1 + 0.0386 X_3 + 0.0281 X_4 - 0.142 X_6 - 0.567 X_7 \quad \text{(9)} \]

\[ Y_G = 23.3 + 0.0193 X_1 + 0.0591 X_3 - 0.0371 X_5 - 0.133 X_6 - 0.593 X_7 \quad \text{(10)} \]

The improvement in \( R^2 \) from equations with five predictor variables over equations with four predictor variables is small so the models (7) and (8) are better for data on grade of copper.

Both the above models are not significantly different from models (7) and (8).

The two best regression equations involving six predictor variables are given below:

\[ Y_G = 22.3 + 0.019 X_1 + 0.039 X_3 + 0.018 X_4 - 0.023 X_5 - 0.130 X_6 - 0.602 X_7 \ldots \ldots \quad \text{(11)} \]

\[ Y_G = 19.8 + 0.019 X_1 + 0.068 X_3 + 0.036 X_3 + 0.028 X_4 - 0.142 X_6 - 0.567 X_7 \ldots \ldots \quad \text{(12)} \]

Both the above models are not statistically significant.

The full model involving seven predictor variables is:

\[ Y_G = 21.6 + 0.0194 X_1 + 0.066 X_3 + 0.0383 X_4 + 0.0182 X_4 - 0.0236 X_5 - 0.130 X_6 - 0.602 X_7 \quad \text{(13)} \]

\( X_2, X_3, X_5, \) and \( X_6 \) are not statistically significant only \( X_1, X_4, \) and \( X_7 \) are statistically significant.

**CONCLUSION:**

From this research study, we have concluded that the best fitted model consists of three variables, that is collector type and dosage \( (X_1) \), sulfidizer sodium sulphide \( (X_4) \), and conditioning time \( (X_7) \). The following is the fitted model:

\[ Y_G = 16.5 + 0.0216 X_1 + 0.0448 X_4 - 0.598 X_7 \ldots \ldots \quad \text{(5)} \]

It is obvious from this model that if we increase one unit of \( X_1 \), \( Y_G \) will increase 0.0216 units keeping all other variable constant. Similarly we can define the other entire coefficient in the same fashion.
Figure 1: Effect of sodium cyanide ($x_3$) on the grade of copper.

Figure 2: Effect of sodium sulphide ($x_4$) on the grade of copper.

Figure 3: Copper grade ($Y_G$) response surface for sodium cyanide ($X_3$) and conditioning time ($X_7$).

The combine response surface for sodium cyanide and conditioning time on the grade of copper is shown in Figure 3. The maximum peak of surface shows the estimated maximum grade of 16.42% with 28 gram per ton of sodium sulphide and 11 minutes conditioning time.

Figure 4: Copper grade ($Y_G$) response surface for sodium sulphide ($X_4$) and conditioning time ($X_7$).

The combine response surface for sodium sulphide and conditioning time on grade of copper is shown in Figure 4. The maximum peak of surface shows the estimated maximum grade of 17.36% with 55 gram per ton of sodium sulphide and 10 minutes of conditioning time.
REFERENCES


Wang., Z. 1996 Lecture on “Distribution and Metallogenic models of large Copper deposits of the world in North Waziristan Copper Ore.